CONTRIBUTION TO THE ANALYSIS OF LIQUID MACROFLOW IN A CYLINDRICAL VESSEL WITH A HIGH SPEED IMPELLER* **

Jaroslav MEDEK[a] and Ivan FOŘT[b]

[a] *Department of Chemical and Food Machinery, Technical University, 602 00 Brno and*
[b] *Department of Chemical Engineering, Prague Institute of Chemical Technology, 166 28 Prague 6*

Received August 8th 1979

In hitherto published studies the investigation of liquid macroflow in a vessel with an impeller has been concentrated mostly on the assessment of the volumetric flow rate through the impeller. Due to molecular and turbulent diffusion, however, significant exchange of mass and momentum occurs between this flow and the neighbouring charge. The extent of this induced volumetric flow rate has not been systematically investigated. This contribution attempts at the assessment of the induced, and total volumetric flow rate components in a vessel, in the plane of rotation of the lower edge of an impeller with flat inclined blades, in dependence on the blade number and the inclination angle between the blade and this plane.

Mixing is still considered to be one of the important chemical engineering operations. In a vessel with a mechanical impeller a rather complex flow pattern occurs, which is difficult to describe mathematically. Therefore in literature the experimental approach still prevails. The application of design rules and conventions proposed without deeper theoretical knowledge for relatively small units has often been found dubious in case of large-scale equipments required by the contemporary industrial trends. Hence, from experimental results general theoretical considerations should also be drawn.

In order to change the empirical character of the mixing equipment design, the problems of the mixing process have to be investigated in their complexity. To obtain the basic information needed, the investigation should be focused on: *1*) the determination of streamline pattern of stirred liquid, in dependence on the type of impeller and the geometry of the mixing equipment; *2*) the determination of volumetric flow rates in various regions of the stirred tank; *3*) the choice of mixing equipment geometry by means of which the required streamline pattern and volumetric flow rate of charge may be attained; *4*) the determination of impeller power input necessary for obtaining the streamline pattern according to *3*. *5*) With respect to the purpose of the mixing process, not only the influence of recirculation flow but also of turbulent and molecular diffusion ought to be investigated.

This study is addressed especially to the second aim in the above list – *i.e.* the assessment of the volumetric flow rates of liquid in the plane of rotation of the lower blade edge of an impeller with flat inclined blades, in dependence on the blade number and their inclination angle with respect to this plane.

* Part LVI in the series Studies on Mixing; Part LV: This Journal *45*, 2152 (1980).
** Delivered at the 3rd European Conference on Mixing, York, 1979.

The determination of streamline pattern in a vessel with a mechanical impeller is dealt within a number of studies, describing sufficiently not only streamline patterns with prevailing axial, radial or tangential flow, but also types of impellers and mixing equipment geometry, by means of which the required flow patterns may be attained[1-3].

The impeller with flat inclined blades (Fig. 1) is a type of impeller creating in a cylindrical baffled vessel a pronounced convective flow of stirred liquid. The geometry of this impeller is characterized by a set of quantities which may significantly affect the hydrodynamic qualities of the impeller. They are: *1*) number of blades n_L, *2*) the inclination angle of blades with respect to the plane of rotation of the lower edges of impeller blades (α), 3) blade width h.

Additionally, further design characteristics may be defined which also influence, though to a smaller extent, the hydrodynamic qualities of the impeller. These are especially: *4*) blade thickness s, *5*) hub and impeller diameter ratio or in some cases hub shape.
Some combinations of the above geometrical characteristics are standardized[4]. This study deals only with the influence of the impeller geometry, neglection the impact of the design characteristics.

The motion of liquid caused by impellers with prevailing axial flow creates a characteristic streamline pattern[5-8] (Fig. 2). In most cases the impeller is designed to direct the liquid flow downwards. In the pattern of circulation loops two main types of streamlines are apparent:

a) streamlines passing through the impeller and forming the so-called primary flow of liquid characterized by the volumetric rate through the impeller $\dot{V}$, *b*) streamlines by-passing the impeller and acquiring momentum by turbulent and viscous friction from the streamlines of primary flow. They are characterized by the so-called induced volumetric flow rate of the impeller $\dot{V}_i$.

Consequently, we may define:

Volumetric flow rate through the impeller $\dot{V}$ (primary flow rate) as the flow rate of liquid from the base and the surface of the cylinder circumscribed by the impeller (so-called rotor region).

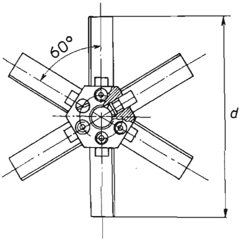

FIG. 1

Impeller with Flat Inclined Blades

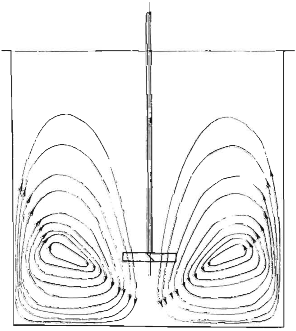

FIG. 2

Streamline Pattern Created by Impeller with Prevailing Axial Flow of Liquid

Total volumetric flow rate in the vessel $\dot{V}_c$ as the flow rate through the horizontal cross-section of the vessel in the level of rotation of lower edges of impeller blades, always in the direction of primary volumetric flow rate through the impeller.

Induced volumetric flow rate in the vessel $\dot{V}_i$ as the part of total volumetric flow rate which does not pass through the rotor region of the impeller.

From the suggested definitions follows:

$$\dot{V}_c = \dot{V} + \dot{V}_i \,. \qquad (1)$$

All volumetric flow rates introduced in this way may be expressed by means of dimensionless criteria:

criterion of volumetric flow rate through the impeller:

$$K_p = \dot{V}/nd^3 \,, \qquad (2)$$

criterion of total volumetric flow rate in the vessel:

$$K_{pc} = \dot{V}_c/nd^3 \,, \qquad (3)$$

criterion of induced volumetric flow rate in the vessel:

$$K_{pi} = \dot{V}_i/nd^3 \,. \qquad (4)$$

Considering relation (*1*), we may write:

$$K_{pc} = K_p + K_{pi} \,. \qquad (5)$$

The source of convective flow in stirred liquid is the volumetric flow rate created by the impeller and characterized by K_p. Consequently, the quantities K_{pc} and K_{pi} are dependent on the volumetric flow rate of liquid through the impeller. Like K_p, they depend mainly on the type of impeller and the geometry of the mixing equipment. This study is focused especially on investigating the influence of impeller geometry on the mentioned quantities in the mixing equipment of constant geometry.

THEORETICAL

The design of the mixing equipment investigated in this study is characterized by a flat-bottomed cylindrical vessel of diameter D. The vessel is filled with a low viscosity newtonian liquid with the height $H = D$. One high speed impeller of diameter $d < D/2$ is situated in the vessel axis. The direction of rotation and the impeller design are chosen so that the axial component of the volumetric flow rate through the impeller[5,7] is directed towards the vessel bottom. Baffles of width $b = 0{\cdot}1$ D fully suppressing the tangential flow component are placed at regular distance alongside the vessel walls, their height being identical with that of the vessel (H) (Fig. 3).

To determine the velocity field in the stream flowing through the total cross-sectional area of the vessel at the level of the lower edges of impeller blades following assumptions have to be introduced: A *1*) In the definition of the parts of check plane (rotor region) impeller design characteristics are neglected. A *2*) The flow regime of agitated liquid is fully turbulent and may be characterized by the Reynolds number value $Re_M \geqq 1.0 \cdot 10^4$. A *3*) In spite of the high value of Re_M the charge is homogeneous and no aeration from the liquid surface occurs, which would change the physical properties of the charge during mixing. A *4*) The streamline pattern of the stirred liquid is symmetrical with respect to the vessel axis coincident with the axis of rotation and the impeller shaft. A *5*) In the investigated volume mutual interaction of impeller blades is neglected. A *6*) The liquid is given momentum only by the pressure of impeller blades.

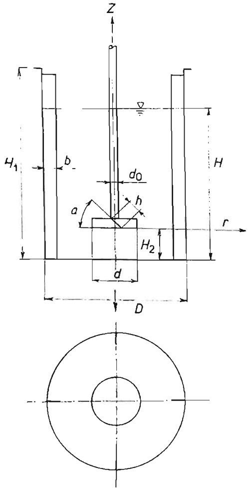

FIG. 3

Mixing Equipment Geometry

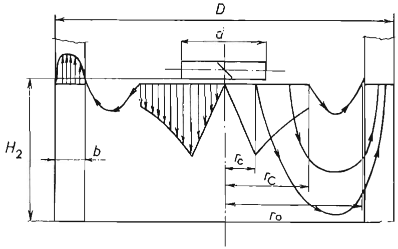

FIG. 4

Velocity Profile in Vessel Cross-Section in Rotation Level of Lower Edges of Impeller Blades

The mean velocity profile in the check-plane may be divided into the following regions (Fig. 4):

1) The cylindrical region of diameter $r = r_c$. Liquid flow is characterized by constant angular velocity[5].

2) The rotational region of inner radius $r = r_c$ and outer radius $r = d/2$. Liquid flow is characterized by the hyperbolic velocity profile. For $r = r_c$ the velocity reaches its maximum and then gradually decreases with increasing radius in the interval $r \in \langle r_c; d/2 \rangle$.

3) The rotational region adjacent to region 2. Its outer part is determined by $r = r_C$. Liquid velocity decreases continuously according to the same relations as in region 2.

4) Because of the final vessel shape the stream of liquid flowing through the check plane turns upwards alongside the vessel walls. The descending and ascending sections are separated by the "region of relative rest". This region may be defined: $r \in \langle r_C; r_0 \rangle$.

5) Ascending region, which can be determined from the continuity equation for incompressible liquid, namely from the condition that the whole liquid volume per unit time leaving the check plane in the direction of the volumetric flow rate through the impeller must flow through the ascending region defined by $r \in \langle r_0; D/2 \rangle$. Mean velocity profile in this region is not considered.

For thus defined regions, the values of local velocity components may be determined from the following equations:

Region 1: $r \in \langle 0; r_c \rangle$

$$\bar{w}_{ax} = \pi n r \sin 2\alpha , \qquad (6a)$$

$$\bar{w}_{tg} = 2\pi n r \sin^2 \alpha . \qquad (6b)$$

Region 2: $r \in \langle r_c; d/2 \rangle$

$$\bar{w}_{ax} = \pi n (r_C^2/r) \sin 2\alpha , \qquad (7a)$$

$$\bar{w}_{tg} = 2\pi n (r_c^2/r) \sin^2 \alpha . \qquad (7b)$$

Region 3: $r \in \langle d/2; r_C \rangle$

$$\bar{w}_{ax} = \pi n (r_C^2/r) \sin 2\alpha , \qquad (8a)$$

$$\bar{w}_{tg} = 2\pi n (r_C^2/r) \sin^2 \alpha . \qquad (8b)$$

Region 4: $r \in \langle r_C; r_0 \rangle$

$$\bar{w}_{ax} \doteq 0\,, \tag{9a}$$

$$\bar{w}_{tg} \doteq 0\,. \tag{9b}$$

Region 5: $r \in \langle r_0; D/2 \rangle$

$$\bar{w}_{ax} = \dot{V}_C/\pi(D^2/4 - r_0)\,, \tag{10a}$$

$$\bar{w}_{tg} \doteq 0\,. \tag{10b}$$

Due to the centrifugal force caused by the rotation of impeller blades the tangential velocity component may be considered equal to the radial component[9]. In region 5 purely axial liquid flow is considered due to the action of the radial baffles at the vessel walls.

The volumetric flow rate through the impeller may be calculated by the integration of the above introduced velocity profile across regions 1 and 2 as:

$$\dot{V} = \iint_{A\mathrm{rot}} \bar{w}(A)\,\mathrm{d}A\,. \tag{11}$$

After substitution and rearrangement we obtain:

$$\dot{V} = \left[\pi^2/12(3c^2 - 2c^3)\sin 2\alpha + \pi^2/5\, c^2 \sin^3 \alpha\right] nd^3\,, \tag{12}$$

where

$$c = 2r_c/d\,. \tag{13}$$

In the brackets on the r.h.s. of Eq. (*12*) the first member represents the contribution of axial flow and the second one the contribution of the radial flow through the rotor region[5,7].

The induced volumetric flow rate can be obtained by similar integration of the velocity profile across region 3 – *i.e.*:

$$\dot{V}_i = \frac{\pi^2}{2} c^2(\cos\alpha \sin\alpha + \sin^3\alpha)(C_d - 1)\, nd^3\,, \tag{14}$$

where

$$C_d = 2r_c/d\,. \tag{15}$$

In processing the results of experimental measurements of the volumetric flow rate through the impeller $\dot{V}$, the values of parameter c may be obtained by solving cubic equation (*12*). Knowing this parameter, the range of the impeller influence on the liquid flowing to the vessel bottom (parameter C_d) may be determined from the experimentally obtained value of induced volumetric flow rate in the vessel. For better understanding it is suitable to introduce parameter C_D (related to the vessel diameter). Between both parameters C_d and C_D, there is a simple relation:

$$C_D = C_d(d/D)\,. \qquad (16)$$

EXPERIMENTAL

For the measurements of the volumetric flow rate through the impeller the method of measuring liquid circulation by means of tracer particle[5,10,11] has been used. The volumetric flow rate through the impeller may be calculated from the following relation:

$$\dot{V} = V/\Theta_1\,. \qquad (17)$$

This method has the advantage that it can be applied to measurements of total volumetric flow rate in the vessel as well, since by analogy:

$$\dot{V}_c = V/\Theta_2\,. \qquad (18)$$

The mean circulation times Θ_1; Θ_2 are always considered as an average interval between two subsequent passes of the tracer particle through the check-plane. For the flow rate $\dot{V}$ the rotor region is considered as the respective check-plane. For the total volumetric flow rate $\dot{V}_c$ the horizontal cross-section at the level of the lower basis of the rotor region is taken as the check-plane.

With respect to the random character of liquid circulation the precision of the evaluation of quantities Θ_1 and Θ_2 is given by the total number of tracer particle passes through the check-planes. In this study, always 2000 passes were followed, this number being considered sufficient[5].

The investigation was focused on the impellers with flat inclined blades with blade numbers $n_L \in \langle 2, 6 \rangle$ and $n_L = 8$ and 10, and blade inclination angle $\alpha = 45°$. In case of the impellers with blade numbers $n_L = 3$ and 6 the dependence of flow rates on blade inclination angles $\alpha \in \langle 15°; 90° \rangle$ was investigated. All experiments were carried out in the mixing apparatus of standard design (according to the data given in Table I).

As a stirring medium water of temperature 20°C was used. With respect to the used size of impellers ($d = 150$ mm) rotation frequency was chosen so that the condition of turbulent

TABLE I
Mixing Equipment Geometry

H/D	H_2/d	D/d	b/D	n_b	h/d
1·00	0·75	3·00	4	4	0·20

regime of agitated liquid ($Re_M < 1 \cdot 10^4$) was always satisfied. The impeller design provided for a continuous change of blade inclination angle.

RESULTS AND DISCUSSION

Using the results of measurements and relations (*12*) and (*13*) the values of universal parameters c and C_d (or C_D), in dependence on the number of blades and blade inclination angle, were calculated, for the impellers with blade numbers $n_L = 3$ and 6. The values obtained in this way are summed up in Tables II and III. The graphical representation of the respective dependences is shown in Figs 5 and 6.

TABLE II

Values of Parameters c, C_d, C_D in Dependence on Impeller Blade Number

n_L	c	C_d	C_D
2	0·564	1·582	0·527
3	0·599	1·522	0·507
4	0·632	1·513	0·504
5	0·660	1·499	0·499
6	0·703	1·439	0·480
8	0·733	1·401	0·467
10	0·723	1·435	0·487

TABLE III

Values of Parameters c, C_D, C_d in Dependence on Blade Inclination Angle for Impellers with Blade Numbers $n_L = 3$ and 6

α	$n_L = 3$			$n_L = 6$		
	c	C_d	C_D	c	C_d	C_D
15°	0·686	1·709	0·570	0·940	1·437	0·479
30°	0·573	1·564	0·521	0·756	1·501	0·500
45°	0·599	1·522	0·507	0·703	1·439	0·480
60°	0·568	1·540	0·513	0·632	1·430	0·477
75°	0·574	1·368	0·456	0·628	1·254	0·418
90°	0·613	1·209	0·403	0·652	1·221	0·407

The course of the function $c = f(n_L)$ (Fig. 5) shows that the value of parameter c increases with the increasing blade number, up to the value of $n_L = 8$. This means that for the given mixing equipment design with constant blade inclination angle (in this case $\alpha = 45°$) the volumetric flow rate increases. On the contrary, parameter C_D characterizing the induced volumetric flow rate can be considered constant with a sufficient precision in the range of the experiments carried out ($n_L \in \langle 2; 10 \rangle$). The mean value C_D may be calculated from all the measurements as arithmetic average. Consequently, in the given conditions the induced volumetric flow rate remains practically constant with blade numbers changing. Hence, the increase of the total volumetric flow rate in the vessel is caused by the increase of the volumetric flow rate through the impeller. The values of C_D also comply with the basic physical notion, according to which such values should be smaller than $\sqrt{2}/2$. This magnitude can be obtained from the continuity equation for incompressible liquid between the ascending and descending flow in the check-plane.

For both impellers investigated ($n_L = 3$ and 6) the dependence of c on impeller

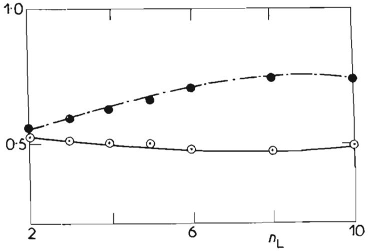

FIG. 5

Dependence of Values c and C_D on Impeller Blade Number

● c; ⊙ C_D.

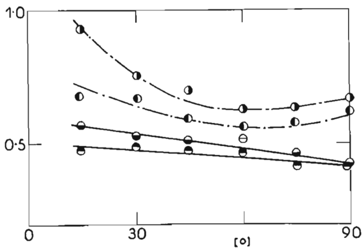

FIG. 6

Dependence of Values c and C_D on Inclination Angle of Impeller Blade α

◑ c for $n_L = 6$; ◒ C_D for $n_L = 6$, ◐ c for $n_L = 3$; ◓ C_D for $n_L = 3$.

blade inclination reaches its maximum at small angles ($\alpha = 15°$). Consequently, at small blade inclination angles, the maximum speed in the velocity profile considered is not reached before the tip of blade, which does not accord with the physical notions (see Assumption A *6*). The more detailed study of this phenomenon has shown that at small inclination angles the circulation of liquid around impeller blade occurs, having a notable influence on the volumetric flow rate of liquid through the impeller[5]. The existence of this component may be easily proved by a simple experiment, if the blades are designed *e.g.* as sections of cylinder shell area, which is known to have greater uplift effect than a flat inclined plane[12]. The experiments carried out with thus adapted impellers have proved higher values of the volumetric flow rate through the impeller at $\alpha = 15°$ than in case of the impellers with flat inclined blades[5].

With increasing blade inclination angle the C_D value decreases for both impellers investigated (Fig. 6). The induced flow rate is produced, above all, by the axial component of the volumetric flow rate through the impeller, which, however, decreases for $\alpha < 45°$[5,7]. This also explains the decrease of C_D with the blade inclination angle increasing. The increase of c for $\alpha < 45°$ is, on the other hand, caused by a significant increase of radical volumetric flow rate through the impeller[5,7]. Eventually, the fact that the C_D values increase for impellers with smaller blade numbers (in the whole range of α investigated) is worth mentioning. In impellers with greater blade numbers greater volumetric flow rate through the impeller occurs. This also causes the increase of total volumetric flow rate $\dot{V}_c$, which, in turn, brings about the growth of the flow rate in the region close to the vessel walls. As long as the value of the induced volumetric flow rate of liquid is independent of the blade number (Fig. 5), the C_D value for the impellers with greater blade numbers must be smaller than for those with smaller blade numbers.

CONCLUSION

The convective turbulent flow of homogeneous viscous newtonian liquid in a cylindrical vessel with baffle-plates and one impeller with flat inclined blades may be, in the horizontal cross-sectional area under the rotating impeller, described by a two-parameter model. The parameter c, characterizing the position of velocity maximum in the considered velocity profile increases with increasing blade number, until it reaches the value $n_L = 8$. On the other hand, the c value decreases simultaneously with the blade inclination angle, up to $\alpha = 60°$. The c value may be calculated from the experimentally found values of the flow rate criterion for the volumetric flow rate through the impeller.

The parameter C_D, characterizing the range of impeller action in the vessel is independent of the number of blades (in the range of $n_L \in \langle 2; 10 \rangle$). It decreases with the blade inclination angle in the whole range investigated. Knowing the volumetric flow rate through the impeller, the C_D value may be calculated from the experimentally found value of the flow rate criterion for the total volumetric flow rate in the vessel.

LIST OF SYMBOLS

A_{rot}	rotor region of impeller (m^2)
b	baffle width (m)
c	dimensionless parameter according to Eq. (*13*)
C_d	dimensionless parameter according to Eq. (*15*)
C_D	dimensionless parameter according to Eq. (*16*)
D	vessel diameter (m)
d	impeller diameter (m)
H	height of liquid in vessel when impeller at rest (m)
H_1	total height of vessel (m)
H_2	distance of impeller lower blade edge from vessel bottom (m)
h	impeller blade width (m)
n	impeller rotation frequency (s^{-1})
n_b	number of baffles
n_L	impeller blade number
r	radius (m)
r_c	limiting radius of region 1 (m)
r_C	limiting radius of region 3 (m)
r_O	limiting radius of region 4 (m)
s	impeller blade thickness (m)
V	volume of stirred liquid (m^3)
$\dot{V}$	volumetric flow rate through impeller (m^3 s^{-1})
$\dot{V}_c$	total volumetric flow rate of liquid in vessel (m^3 s^{-1})
$\dot{V}_i$	induced volumetric flow rate of liquid in vessel (m^3 s^{-1})
$\overline{w}$	liquid mean velocity (m s^{-1})
α	inclination angle of impeller blade (°)
Θ_j [$j = 1, 2$]	average time of tracer particle circulation (s)
η	dynamic viscosity (kg m^{-1} s^{-1})
ϱ	density (kg m^{-3})
Kp	flow rate criterion of volumetric flow rate through impeller
Kp$_c$	flow rate criterion of total volumetric flow rate of liquid in vessel
Kp$_i$	flow rate criterion of induced volumetric flow rate of liquid in vessel

$\text{Re}_M = d^2 n \varrho \eta^{-1}$ Reynolds number

REFERENCES

1. Uhl V. W., Gray J. B.: *Mixing — Theory and Practice*, Vol. I. Academic Press, New York and London 1966.
2. Nagata S.: *Mixing — Principles and Applications*. Kodansha Ltd., Tokyo 1976.
3. Strek F.: *Míchání a míchací zařízení*. Published by SNTL, Prague 1977.
4. Czechoslovak Standard "Mixing" ON 691000 — 37. VÚCHZ — Chepos, Brno 1969.
5. Medek J.: *Thesis*. Technical University Brno, Brno 1976.
6. Medek J., Fořt I.: *Convective Flow in a Cylindrical System with Radial Baffles and Paddle Impeller with Inclined Plane Blades*. 3d European Conference on Mixing, York 1979.

7. Medek J., Fořt I.: This Journal *44*, 3077 (1979).
8. Porcelli J. V., Marr G. R.: Ind. Eng. Chem., Fundam. *1*, 182 (1962).
9. Vusse de Van J. G.: Chem. Eng. Sci. *4*, 202 (1955).
10. Fořt I., Valešová M., Kudrna V.: This Journal *36*, 164 (1971).
11. Medek J., Fořt I.: Chem. Prům. *29*, 62 (1979).
12. Nechleba M.: *Hydrodynamika lopatkových strojů*. Technical University Brno, Brno 1968.

Translated by M. Procházka.